Structure of Indian *Rāga*s: *Mātrkā*, the Fifth Note and *Shruti*

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Abstract

While it is very difficult to describe a *Rāga* using words or by specifying a few of their salient features, there is no doubt that there is something very profound and natural about their existence. A *Rāga* is not merely a scale, neither it is a mood, however, an expert performer can improvise freely while staying within the periphery of a *Rāga.* Sanyal [1] analyzed many faithful specimens of a variety of *Rāga*s and came to the conclusion that a particular structure of four notes, what he called *Mātrkā,* is at the root of a *Rāga* formation. Sanyal also introduced the concept of a fifth note, however, its role remained vague in his work. In this paper, from idealized considerations of the Harmonic Scale, Sanyal's basic proposal of *Mātrkā* is validated. It is also argued that an appropriate fifth note is essential for making a stable melodic structure out of several possibilities from a particular *Mātrkā*. A significant advantage of the present work is that, ideal pitch of all the notes in a given *Rāga* can be explicitly determined and the microtonal variations (*Shruti)* understood. Examples are given with *Rāga*s *Bhoopāli*, *Shuddha Kalyān*, *Yaman*, *Megh*, *Madhyamādi Sārang*, *Mārwā*, *Puriyā*, *Sohini*, *Tori*, *Multāni*, *Bhairav* and *Rāmkeli*. It is to be noted that Sanyal's work or its development in the form of the present paper should be regarded as a beginning of a new way of thinking about the basic foundations of a *Rāga* and is far from a last word on the subject.

Keywords: Harmonic Scale, *Mātrkā*, *Rāga*, *Shruti*

1. Introduction

*Rāga*s are melodic structures in Indian music that are passed on to the disciples by the Gurus over the generations. Majority of available literature describes a *Rāga* by listing a few of its salient features: for example, each *Rāga* has a particular set of basic notes in the Octave having one principal note (called *Vādi*), another equally important note (called *Samvādi,* having a perfect fifth or perfect fourth relationship to the *Vādi*) and several other notes (called *Anuvādi*), an ascending and a descending pattern, and sometimes also a few other rules for movement from one note to another (called *Chalan*).

In the last few centuries, there have been attempts to classify the *Rāga*s based on *Janaka-Janya Mela* (or *Thāt*) system [2,3]. For example, let us start with a parent (*Janaka*) scale (*Mela)* called *Kalyān or Kalyāni*: **SRGmPDN** (please see Appendix for an explanation of the notation used). Drop the notes **m** and **N** and we get the notes of the derived (*Janya*) *Rāga Bhoopāli*. Alternatively, by dropping the notes **m** and **D**, we get the notes of *Rāga Hamsadhwani*; or drop the notes **R** and **P** to get the notes of the *Rāga Hindo*l. If we keep all the notes, we get the notes of the *Rāga*s *Yaman* and *Shuddha Kalyān*. This classification system has sometimes also been described as a theory of the Ragas.

A *Rāga* is taught, however, through mostly oral traditions which do not use a lot of the notions given in the above two paragraphs. The student learns it by repeating the phrases sung or played by the Guru, and afterwards the basics of the *Rāga* get so internalized that the student-turned-performer can freely improvise the movements while staying within the boundary of the *Rāga.* Using a metaphor, the exposition of a *Rāga* is something like a tram-car moving along a particular track – as long as the operation is within the mechanical limits, the tram never gets derailed. Similarly, despite free improvised movements, the expert performer never gets out of the melodic structure of a given *Rāga.* Each *Rāga* appears to have a robust nature such that spontaneity of the performer cannot destabilize the basic melodic structure and the listener can identify the *Rāga* unambiguously.

The information in the first two paragraphs of this section appears ad hoc and does not explain the genesis of a *Rāga.* In addition, the *Janaka-Janya* (or the *Mela*) system is logically unsound, as, for example, notes of the *Rāga Bhoopāli* can be obtained by the deletion of appropriate notes from several parent scales or *Mela*s, (**M** and **N** from *Bilāwal* (**SRGMPDN**), **M** and **n** from *Khamāj* (**SRGMPDn**), and **m** and **N** from *Kalyān* as mentioned above).

In this paper, a completely different approach is adopted to understand what constitutes a *Rāga*. Why should a *Rāga* be derived from a parent scale? The proposition in this paper is that, given the robustness of a *Rāga* in the sense described above, a *Rāga* must have an independent identity with a very distinctive personality and any pitch not contained in the scale of the *Rāga* is completely irrelevant. For example, for *Rāga Bhoopāli*, it is as if anything other than the notes **SRGPD** does not exist in the universe. Imagine a universe where there are no fixed-pitch instruments like keyboards. Then there is no direct physical awareness of the presence of any extra notes. There is only a reference pitch (or tonic which we call *Sadaja* in the Indian musical parlance) with respect to which the other notes in a given melody are generated, the reference pitch being produced by a drone instrument called *tāmbourā* or *tānpurā*.

Amiya Nath Sanyal in his little known but substantial work [1] took the first steps in this novel approach. This approach was later discussed and examined in [4]. Sanyal made a statistical analysis of many specimens in a given *Rāga* and then repeated the exercise on a multitude of other *Rāga*s. His conclusions were drawn from the values of a few clearly defined parameters resulting from the analysis. He observed a particular melodic structure appearing in each familiar specimen of a *Rāga.* The current work examines Sanyal's work from the point of view of the principles of generation of the so-called Harmonic Scale and establishes how *Shruti* or microtonal variations of the same note can appear depending on the context[1] the note is taken and how the ideal pitch of every note in a *Rāga* can be determined.

There is a tendency among some of the modern musicologists, mostly from the Western world, to dismiss the microtonal variations or the *Shruti*s as a myth [5]. However, there is no doubt that every experienced performer of *Rāga* music strives to reach the ideal intonations for each *Rāga*. We shall come back to this point at the end of the paper, in the Conclusions.

2. Harmonics and Emergence of a Melodic Structure

Let us start with an example. Several performers have sung the same folk/devotional/regional song (perhaps with minor differences to the tune) over the years, and generally they have sung it using different tonics (called the *Sadaja* in the Indian context). However, barring the obvious differences due to the tonal qualities and expertise of the singers, the general effect of the tune is the same. This immediately shows that the melody in the Indian context is a particular sequence of notes where the absolute pitch (frequency) of the notes is not relevant, rather their relative pitch with respect to each other and to the tonic is important, and the pitch of the tonic may vary according to the choice of the singer. This is unlike Western music where in current practice the absolute pitch is assigned an important role. But in the Indian context, one may say that the note "Do" is variable. The other important inference, although a derived and indirect one, is that the human sensory organs which receive and appreciate the music, keeps in memory the past notes used in the sequence [6], because the next note in the sequence has to fulfill the demand created by the previous notes.

From this discussion it is now apparent that all the notes used in a melodic sequence is defined with respect to the *Sadaja* and the *Sadaja* has an arbitrary pitch. Now the obvious question is: how are the pitches of the other notes used in a melody determined? Is there a natural method or is it imposed by an arbitrary or ad-hoc proposition?

We shall use a scientific input [7] at this point from the field of Acoustics: *What we call a single-pitched musical sound is never composed of a single frequency. It is a superposition of several components each having a different frequency. There is a component with the lowest frequency (called the fundamental frequency, responsible for the pitch of the sound) and usually having the largest energy content. There exist several other frequency components known as the harmonics.* By definition, the first harmonic is the same as the fundamental frequency and the second and higher harmonics have frequencies which are integer multiples of the fundamental. For example, a musical sound having the fundamental frequency f will generally have higher harmonics with frequencies nf for the n-th harmonic. Based on the above, the Octave relation (the basic concept in all music, called *Saptaka* in the Indian context) is defined between two notes having a ratio ***2:1*** of the fundamental frequencies. The harmonics of the note with pitch f are $f, 2f, 3f, 4f, 5f, 6f, 7f, 8f$ etc and those of the note with pitch $2f$ are $2f, 4f, 6f, 8f$ etc, that is, every harmonic of the note in the higher Octave coincides with every *even* harmonic of the note in the lower Octave. Hence, if these two notes are played or sung at the same time or in sequence, they sound pleasant and are said to be in tune. This is the highest degree of consonance between two notes. In the following, we also use the word ‘Consonance’ to mean a specific pleasing relationship[2].

By a similar argument, the pitch ratio ***3:2*** of two notes is pleasant and is known as a Consonance (*Samvāda*)

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[1] Here the word ‘context’ means the immediately preceding notes.

[2] The word ‘Consonance’, with a capital ‘C’ for the first letter, refers specifically to the perfect fifth or perfect fourth relationship (*Samvāda*). In general, two notes whose pitch are related by ratios of small integers are said to be in consonance.

relation, because the harmonics of the higher pitched note in this case are *3/2f, 3f, 9/2f, 6f, 15/2f, 9f* etc and we see that every *even* harmonic coincides with *every third* harmonic of the lower-pitched note. Played together or in sequence, therefore, they sound pleasant and in tune. Similarly, the pitch ratio ***4:3*** is also essentially a *Samvāda* because the pitch *2f* at the higher Octave is *3/2* times the fundamental frequency of the pitch at *4/3f*.

In Indian notation, if we call the pitch at *f* the *Sadaja* (**S**) of the middle Octave (*Madhya Saptaka*), the pitch at *2f* is called the *Sadaja* of the higher Octave (*Tāra Saptaka*), the pitch at *3/2f* is called the *Panchama* (**P**), and the pitch at *4/3f* is called the *Madhyama* (**M**). As explained above, the *Sadaja* of the *Tāra Saptaka* has a pitch which is *3/2* times that of the *Madhyama* and that is why this is also a *Samvāda* relationship.

Continuing along the same line, the pitch ratio ***5:4*** of two notes is also pleasant and is known as a major third (*Anuvāda*) relation because the harmonics of the higher note are *5/4f, 10/4f, 15/4f, 5f, ..., 10f* etc and *every fourth* harmonic coincides with *every fifth* harmonic of the lower note. In Indian notation, if the pitch *f* is called the *Sadaja*, the note at pitch *5/4f* is called *Shuddha Gāndhāra* (**G**). Using similar logic of harmonics, the note at pitch *6/5f* is also a pleasant note in relation to the *Sadaja* at pitch *f* and is known as *Komala Gāndhāra* (**g**) and the relationship of the notes with pitch ratio of ***6:5*** is called a minor third (another *Anuvāda relation*). We should note that if the note **G** is made the tonic (*Sadaja*), the note **P** becomes a minor third, and if the note **g** is made the tonic (*Sadaja*), the note **P** becomes a major third. The **S-G-P** and the **S-g-P** relations are respectively called a major and a minor triad. The previous sentence means that while **S-G** is a major third relationship, **G-P** is a minor third relationship, and similarly while **S-g** is a minor third relationship, **g-P** is a major third relationship. In this sense, the pitch ratios **5:4** and **6:5** are at the same level of consonance.

So far, based on the theory of harmonics, we have discovered increasingly less consonant relationships between two notes in terms of their pitch ratios such as ***2:1***, ***3:2***, ***4:3***, ***5:4*** and ***6:5***. Already we see that a consonance at the pitch ratio **6:5** means that every sixth and every fifth harmonic of the two notes have to coincide. Ratios of bigger integers would require even higher harmonics for coincidence, and as a result, within the range of harmonics produced by musical instruments or human voice, there would be only a few harmonics in consonance and a large number of harmonics in dissonance.

The whole development so far in this section is dependent on the assumption that, consonant relations of various degree between two notes makes the two notes, taken in succession or simultaneously, sound naturally pleasant. This is because some of the higher harmonics of the notes are coincident giving the sense of being in tune. Based on experience in tuning acoustic instruments, this appears to be a plausible assumption.

A scale of notes within an Octave formed using the Consonance and the major or minor third relationships is known as a Harmonic Scale (or Just Intonation).

Let us now try to build a melodic structure within an Octave from the natural concepts described above. A single triad, e.g., a major triad **S-G-P** covers only a part of the pitch spectrum within an Octave. Moreover, it is also not stable, because the note **G** through its higher harmonics will imply the Consonant **N** (*Shuddha Nishāda*). This also means that the minor third relation **G-P** will look for completion of the minor triad **G-P-N**. Hence we now have an interesting structure **S-G-P-N** with pitches of the notes as **S***: f,* **G***: 5/4f,* **P***: 3/2f,* **N***: 15/8f and* **G** and **P** are the two notes mediating between the major and the minor triads. The pitch of the note **N** is obtained by using Consonance with **G**, i.e., by multiplying that of **G** by a factor 3/2. The structure **S-G-P-N** now covers the pitch range of the Octave quite well. However, it is still not guaranteed that this will be a stable structure, because the same argument that was used to extend the structure from **S-G-P** to **S-G-P-N** could be applied at either end to form structures like **S-G-P-N-Ṙ** or **Ḍ-S-G-P-N** etc. In Section 4, a fifth note will be introduced to the four-note structure in a particular way and that appears to bring about the stability by choosing a certain melodic direction.

Instead of starting with a major triad, one can start as well with a minor triad **S-g-P**, and following a similar line of reasoning as above, we end up with the structure **S-g-P-n**. One can carry out this process starting from each nominal note of the Octave. In the following, we list all such structures possible:

1. **S-G-P-N**	2. **S-g-P-n**	3. **r-M-d-Ṡ**	4. **r-G-d-N**	5. **R-m-D-ṙ**	6. **R-M-D-Ṡ**
7. **g-P-n-Ṙ**	8. **g-m-n-ṙ**	9. **G-d-N-ġ**	10. **G-P-N-Ṙ**	11. **M-D-Ṡ-Ġ**	12. **M-d-Ṡ-ġ**
13. **ṃ-ṇ-r-M**	14. **ṃ-Ḍ-r-G**	15. **Ṗ-Ṇ-R-m**	16. **Ṗ-ṇ-R-M**	17. **ḍ-S-g-P**	18. **ḍ-Ṇ-g-m**
19. **Ḍ-r-G-d**	20. **Ḍ-S-G-P**	21. **ṇ-R-M-D**	22. **ṇ-r-M-d**	23. **Ṇ-g-m-n**	24. **Ṇ-R-m-D**

All the odd-numbered structures above are combinations of first a major triad and then a minor triad. The even-numbered structures have first a minor triad and then a major triad. However, they are all *different* structures because the notes are all defined with respect to the same tonic (*Sadaja*).

Writing the above in one Octave and including the *Sadaja* wherever it is not explicit in the structure, we find the following scales[3]:

1. **SGPN**	2. **SgPn**	3. **SrMd**	4. **SrGdN**	5. **SrRmD**	6. **SRMD**
7. **SRgPn**	8. **Srgmn**	9. **SgGdN**	10. **SRGPN**	11. **SGMD**	12. **SgMd**
13. **SrMmn**	14. **SrGmD**	15. **SRmPN**	16. **SRMPn**	17. **SgPd**	18. **SgmdN**
19. **SrGdD**	20. **SGPD**	21. **SRMDn**	22. **SrMdn**	23. **SgmnN**	24. **SRmDN**

3. The Work of Sanyal and *Mātrkā*

Sanyal in 1959 published a phenomenological analysis [1] that was an abridged version of a much larger work. Sanyal studied the opening movement (*Sthāyee*) of a number of authentic compositions in a variety of *Rāga*s and basically counted the time units spent on each of the notes used in the *Rāga*. Sanyal found that the most used notes (i.e. notes at which most time was spent) in every faithful specimen of a *Rāga* showed exactly the structure presented in Section 2, i.e., a four-note structure made of a major and a minor triad interspersed with each other. Sanyal called the structure *Mātrkā*. As hinted in his publication, he borrowed the terminology from ancient scriptures, but did not mention the specific reference. Let us not dwell on the nomenclature and henceforth call this basic melodic structure as a *Mātrkā*, meaning the little mother who keeps in her womb the roots of a *Rāga*. Different *Rāga*s in general originate from different *Mātrkā*s. However, a given *Mātrkā* can give rise to more than one *Rāga*, and we shall present a few examples of this kind in the following sections. Sanyal also found that there are *Rāga*s having more than one *Mātrkā*s, one dominant and the other(s) sub-dominant.

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[3] The sense in which the word 'scale' is used here is as follows. A scale is a collection of notes in a single octave where the notes appear in an ascending order of their respective pitch. A scale is not a piece of melody. it just describes all the notes that may be used in whatever order to produce the melody.

4. The Fifth Note and *Shruti*

Take for example the *Mātrkā* **Ḍ-S-G-P**. Given the pitch of the *Sadaja* **S**, it uniquely determines the pitch of the other three notes: **P** at *3/2*, **G** at *5/4*, and **D** at *5/3 (from now on, we express the pitch of a note by the ratio of its pitch to that of the Sadaja)*. The pitch of the middle Octave **D** is determined by its Consonance (*4:3*) with **G**. From the analysis of Sanyal, a fifth note is always found in *Rāga* specimens such that it is placed between the mediator **S-G** of the two triads and is a Consonant with either of the end-notes of the *Mātrkā* (i.e. **D** or **P** in this case). Sanyal calls this the neutral note because he assumes that the fifth note, in this specific example, is Consonant to both the end-notes **D** and **P**. However, we find that such a note does not exist - a single pitch simultaneously Consonant to both the end-notes cannot exist in general. In this specific example, if we consider the fifth note to be Consonant with **P**, we get a pitch of (*3/*4×*3/2*) = *9/8*, to be called *Shuddha Rshava* (**R**). We have now got the pitch of all the notes present in *Rāga Bhoopāli* and they have all arrived quite naturally, following only the few consonance relations that are basically the requirement of being in tune. Of course, there are other *Rāgas* that can originate from the same *Mātrkā*, but the proposition is that the fifth note **R** at pitch *9/8* makes the note Consonant only with P and thereby pushes the structure to a particular stable direction called *Bhoopāli*. This is a stable direction because the introduction of the fifth note does not give rise to a new triad or *Mātrkā*.

There is certainly another direction possible, if we instead choose the fifth note to be Consonant with the note **D** resulting in a pitch (*2/3*×*5/3*) = *10/9* also called *Shuddha Rshava* (**R**) but obviously flatter than the one that is Consonant with the note **P**. This is a stable direction too because this fifth note also does not introduce a new triad or a new *Mātrkā*. In addition, we clearly see how microtonal variations (*Shruti*) of the same note can appear quite naturally. From careful examination of faithful specimens, it appears that this choice of a flatter **R** (with the **R-D** Consonance) is adopted in *Rāga Shuddha Kalyān.* However, in this *Rāga*, two other notes (*Teevra Madhyama* **m** and *Shuddha Nishāda* **N**) are also used, albeit weakly (only in descending pattern and in traces). Lo and behold! We now discover a second (sub-dominant) *Mātrkā* **Ṇ-R-m-D** in *Rāga Shuddha Kalyān*, where **R-m-D** is a major triad and **Ṇ-R-m** is a minor triad. From the major third relationship **R-m**, the pitch of the new note **m** can be determined to be (*5/4*×*10/9*) = *25/18*, and from the **Ṇ-m** Consonance the pitch of the note **N** is found to be (*4/3*×*25/18*) = *50/27*. We note, with an element of surprise, that, the notes **G** and **N** are not Consonant with each other in *Rāga* Shuddha *Kalyān*. Indeed, faithful renditions of this *Rāga* do not reveal any relationship between its *Gāndhāra* and *Nishāda,* unlike in *Rāga*s such as Yaman.

One may wonder, given the **P̣-R** Consonance in *Rāga Bhoopāli*, why cannot there be similar addition of notes around this Consonance so that a second *Mātrkā* is formed? Let us explore this possibility. Actually, there are several possibilities, and we shall discuss the one closest to the one in *Rāga Shuddha Kalyān.* Let us add the notes **N** and **m** so that a new *Mātrkā* **P̣-Ṇ-R-m** is formed where **P̣-Ṇ-R** is a major triad and **Ṇ-R-m** is a minor triad. By the major third relationship to **P**, the pitch of **N** is obtained as 5/4×3/2 = 15/8 and hence the pitch of **m** by its Consonance to **N** is 3/4×15/8 = 45/32. Firstly, we note that both these notes (**m** and **N**) are sharper than the corresponding notes in *Rāga Shuddha Kalyān.* Secondly, and very significantly, unlike in *Rāga Shuddha Kalyān*, the notes **G** and **N** are now in perfect Consonance, something that is bound to be manifest in exposition of the melody. In addition, another new *Mātrkā* **S-G-P-N** is also formed and will make its presence felt. As a result, the structure would approach that of *Rāga Yaman.*

We find that while in *Rāga Shuddha Kalyān* we can perturb the main structure by softly adding two new notes and this does not change the basic character of the *Rāga* (several versions of this *Rāga* use the **N** and **m** to varied degree and some do not use them at all), in *Rāga Bhoopāli* addition of **N** and **m** changes the character of the *Rāga* in a significant way and brings it close to *Rāga Yaman.* Hence in *Rāga Bhoopāli*, there is no scope of addition of these notes without fundamentally changing the character of the *Rāga.* All of the above is a consequence of choosing the fifth note Consonant with either **D** or **P**, the end notes of the *Mātrkā* **Ḍ-S-G-P**.

5. Other Examples

We start this section with a special *Mātrkā* **P̣-ṇ-R-M**. Firstly, in this structure, the *Sadaja* is not present (there are a few other *Mātrkā*s not having the *Sadaja* as one of the four notes). Secondly, this is the only example where the only choice for the fifth note is *Sadaja* and that is Consonant with both the end-notes **P** and **M** of the *Mātrkā*. The scale is then a pentatonic **SRMPn** giving rise to *Rāga*s *Megh* and *Madhyamādi Sārang*.

Next, we consider the *Mātrkā* **ṃ-Ḍ-r-G**, where again the *Sadaja* is not explicitly present in the structure. Sanyal rejects this *Mātrkā* on the ground that the triad **Ḍ-S-G** is effectively also present, because the tonic note **S** has to be included in the scale[4]. However, I do not see any solid reason not to include this *Mātrkā* because this structure is independent of all others even in the presence of the triad **Ḍ-S-G**. Actually I find it one of the most interesting *Mātrkā*s. There are two choices for the fifth note **N**, either Consonant with the note **m** or with the note **G**. It is a bit of work to figure out the pitches in this example. One needs to start with the note **G** and then use Consonance or the third relationship progressively to figure out the pitches of all the notes progressively: **G** (*5/4*) → **D** (*4/3×5/4 = 5/3*) → **m** (*5/6×5/3 = 25/18*) → **r** (*3/4×25/18 = 25/24*). For *Rāga Mārwā* arising out of this *Mātrkā*, the fifth note **N** appears to be Consonant with **m** to have the pitch (*4/3×25/18*) = *50/27*, and for *Rāga Puriyā*, another *Rāga* to emerge from this *Mātrkā*, the fifth note **N** appears to be Consonant with **G** to have the pitch (*3/2×5/4*) = *15/8*, the sharper microtonal variation. From the authentic specimens and faithful renditions of the *Rāga Mārwā*, we know that the note **G** is quite weak in that *Rāga*, and a movement like **ṃṆḌ**, **ṃḌ**, **ṆṃḌ**, **ḌṆr**, **rṆḌ**, **ḌṆGr**, **mGr** etc de-emphasizes the triad **Ḍ-S-G** and hence this triad cannot cause a de-stabilizing effect. The trick to a good exposition of *Mārwā* lies in the emphasis of the triad **ṃ-Ḍ-r** (not necessarily only in the lower Octave) with the use of **N** in passage and only occasional use of the note **G** (that too only the *Avarohi Meend*s to come back to the note **r**). As always, the proof is in the result, the *Mātrkā* **ṃ-Ḍ-r-G** gives rise to stable and independent melodic structures like *Mārwā* and also *Puriyā*. In *Rāga Puriyā*, we find that the note **G** is very prominent along with its Consonant **N** (sharper than the **N** in *Mārwā* as seen above), and the note **D** is very weak, again de-emphasizing the **Ḍ-S-G** triad. However, in *Rāga Sohini* employing the same set of notes, both **D** and **G** are strong and the triad **Ḍ-S-G** would have its presence felt. The fifth note **N** should logically have the same sharper microtonal variation as in *Puriyā* because the strong presence of **G** imposes the Consonance **G-N**.

As a fascinating example of *Rāga*s having more than one *Mātrkā*s, in both the *Rāgas Tori* and *Multāni*, as is evident from faithful representations of these *Rāga*s, the dominant *Mātrkā* is **ḍ-S-g-P** and the sub-dominant one is **ḍ-Ṇ-g-m**. In *Tori*, the triad **ḍ-S-g** is more important than **S-g-P** and in *Multāni* the triad **S-g-P** is clearly more important, because **P** is more important in *Multāni* than **d** and vice versa for *Tori*. There can be (and actually are claimed in a few *Gharānā*s) several minor variations of *Tori* depending on how much importance is given to the sub-dominant *Mātrkā* **ḍ-Ṇ-g-m** or in other words how important are the notes **N** and **m**, but there is no doubt from the authentic *Vandish* specimens available in *Multāni* that this sub-dominant *Mātrkā* is more important in this *Rāga* than in *Tori*. Now, in *Tori* the fifth note **r** can be fixed by using the Consonance with the note **d** (which itself is fixed by its major third relationship with the tonic note **S**). However, in *Multāni*, the *Shruti* of the note **r** can very well be fixed by the Consonance with the note **m** (end-note of the sub-dominant *Mātrkā* **ḍ-Ṇ-g-m**, the *Shruti* of **m** is fixed by minor third relation with the note **g**, and **g** being fixed again by its minor third relationship from the tonic **S**). Following the two processes, we arrive at two different *Shruti*s for the note **r**. From the overall discussion, we also see the logic behind the

[4] Sanyal rejected in total 8 of the 24 *Mātrkā*s, namely No. 7, 9, 10, 14, 18, 19, 21 and 22 because including the presence of the Sadaj to these *Mātrkā*s imposes a new triad to the structure in each case. The specific case of *Mātrkā* No.14 is taken up here in the discussion of the *Rāga*s *Mārwā, Puriyā* and *Sohini*. A general discussion is out of the scope of this paper and needs a separate discussion.

*Sangati*s **r-g** in *Tori* and **m-g** in *Multāni*. For the record, the pitch of the fifth note **r** in *Rāga* Tori is obtained as follows: **g** (*6/5*) → **d** (*4/3×6/5 = 8/5*) → **r** (*2/3×8/5 = 16/15*), and that in *Rāga Multāni*: **g** (*6/5*) → **m** (*6/5×6/5 = 36/25*) → **r** (*3/4×36/25 = 27/25*). We note that the note **r** in *Multāni* is sharper than that in *Tori*.

Both the *Rāgas Bhairav* and *Rāmkeli* (and also *Kālengrā*) have the notes **SrGMPdN**. From his analysis of many authentic compositions, Sanyal finds that the dominant *Mātrkā* for both is **r-M-d-Ṡ**, and the sub-dominant one is **S-G-P-N**. This is a curious example because all the notes in the scale are consumed by the two *Mātrkā*s. There does not seem to be any room for the fifth note in each *Mātrkā*. However, the note **P** in the sub-dominant *Mātrkā* acts as the fifth note in the dominant *Mātrkā*. Similarly, the note **M** also acts as the fifth note for the sub-dominant *Mātrkā*. In *Rāga Rāmkeli*, it is apparent from the authentic compositions that the role of the sub-dominant *Mātrkā* **S-G-P-N** is more important. However, in *Rāmkeli*, to keep focus on the dominant *Mātrkā* **r-M-d-Ṡ** and especially on the dominant triad **r-M-d**, touching notes (also called grace notes) **m** and **n** are used with the prominent notes **P** and **d** respectively to give rise to a new *Mātrkā* **ṃ-ṇ-r-M**. The pitch of all the notes can be worked out as before and this as well as a discussion on *Kālengrā* is not presented here because of lack of space.

6. Conclusions

This work is inspired by the pioneering work of Sanyal [1] that presented the basic concepts of the *Mātrkā* and the fifth note. The current work, however, has a few significant differences with parts of Sanyal's work. Sanyal conceived the fifth note but its real significance was not understood because he did not consider pitch determinations of the notes from the harmonic relations and dismissed any discussion on the microtonal variations (*Shruti*) of notes. He also rejected 8 *Mātrkā*s from the list of 24 given in Section 2 that do not explicitly include the Sadaja, but an extra triad is implied with the Sadaja included in the scale. In this paper, a specific case is discussed in relation to *Rāga*s *Mārwā, Puriyā* and *Sohini*, but the issue deserves a separate discussion and more work is necessary too.

Let us conclude by considering the following question: Quite a few of the Western composers in the recent times have, at least partly, abandoned the so-called Tempered Scale and composed music employing the Harmonic Scale using relationships such as Consonance and triads. In which way then is the Indian melodic structure called *Rāga* different from those Western compositions?

As emphasized in the Introduction, a *Rāga* is a robust melodic structure, meaning that the structure is stable against spontaneous movements (improvisation) or even weak perturbations. Spontaneous movements do not take one structure to another, one stays within one *Rāga* despite free movements. Even, small perturbations and deviations from the structure do not take it away, as exemplified by the form *Thumri*. A *Thumri* in *Rāga* Kāfi may have elements of independent *Rāgas Pilu*, *Sindhurā*, *Barwa* and a few others but would still at the end of the day be recognizable as *Kāfi*. This is the distinguishing feature of a *Rāga* which makes it very different from just any composition using the Harmonic scale.

Mātrkā is the peculiar construct having the roots of a stable structure: the trick is that it recognizes that a triad by itself is an incomplete structure (for example, a major triad has in itself both the major third and the minor third relationships like **S-G** and **G-P** respectively in **S-G-P**) and hence it needs to be *completed* by having an overlapping triad (**G-P-N** in the above example). A particular *Mātrkā* gives rise to several melodic solutions as manifested by the several *Rāga*s originating from it. In this sense, the *Mātrkā* is also not a completely stable structure. The choice of the pitch of the fifth note in general determines one particular stable melodic direction. This gives the most basic and purest structure, a pentatonic *Rāga*. However, building from the basic structure, more complicated constructions are possible with more than one *Mātrkā*s, as shown in the previous two sections.

We also find that the microtonal variations (*Shruti*) of notes play a very important role, something that Sanyal did not consider at all. It is also obvious that the correct pitch of a note is achieved only if it is taken during performance in a particular sequence, because the sequence of the notes cannot be arbitrary, in fact only the correct sequence ensures the correct pitch: for example, from the *Sadaja* how can one naturally go to a pitch of **N** (*15/8*) because of a very low degree of consonance between the Sadaja and the Nishada. But if this note is taken from **G** (*5/4*), it is very natural, because **N** (*15/8*) is Consonant (*3:2*) to the **G** (*5/4*).

We have been careful to call the mathematically obtained pitch of the notes as *ideal* in this paper. In practice, it is obvious that even with the great musicians, it is very hard to reach these ideal pitch positions for a *Rāga.* Actually, it is a life-long endeavor of each performing *Rāga* musician to achieve what is conceived as the perfect pitch of the notes in a given *Rāga*. The question naturally arises as to how *Rāga* music is feasible in practice, given that ideal pitching is necessary for its manifestation. In the opinion of this author, human perception plays a significant role in bridging some of the gaps, provided some of the essential conditions are met. The analysis given in this paper can help clarify some of these essential conditions. One very important point of this paper is that all the essential conditions are not understood unless one considers the ideal pitch of the notes, as done in this paper, for example to distinguish between *Rāga*s *Bhoopāli* and *Shuddha Kalyān* using the fifth note **R** and its two different pitch positions for the two *Rāga*s**.** The notes immediately preceding **R** determine the expectation according to the rules of melody as discussed in this paper and even if the pitching is not absolutely perfect, the brain accepts the little imperfections in its stride. The hysteresis mentioned in [6] is very important in this regard.

The main reasons behind the criticism of the existence of microtonal variations appear to be inconsistencies between ancient literature and intonations by modern practicing musicians, and variations in intonation of the same note in a given *Rāga* [5]. However, the critics of the *Shruti* system have not even attempted to answer the basic question asked in this paper, namely, if the intonations are really random, what makes every *Rāga* such a stable melodic structure that enables the performer to improvise on the spot and still makes a *Rāga* identifiable by merely listening to it without taking recourse to measurements of the pitch of every note. One needs to understand, for example, despite the presence of both the notes **G** and **N**, why they are not Consonant to each other in *Rāga Mārwā*, making it perfectly feasible for the note **G** to be weak in that *Rāga*. It is, therefore, expected that the pitch of **G** in *Mārwā* may be variable, more than usual, in the same performance or among different performers. In addition, as mentioned in the paragraph above, perfect pitching of even the notes that are in the main melodic framework of the *Rāga***,** for example, the notes **m**, **D**, **r** in *Mārwā* and the note **N** that is Consonant with **m**, is impossible in reality. This is where human perception of the approximate intonation is important. Given that the essential elements of the melodic structure are met by arriving at the pivotal notes after an appropriate sequence or proper use of the grace notes, human brains must perceive the ideal pitch of notes for a given *Rāga.*

There are a few *Rāga*s like *Hindol* that do not exhibit a *Mātrkā* (*Hindol* has a single triad only). Sanyal [1] has discussed these cases. These *Rāga*s, although only a few in number, undoubtedly appear as a self-sufficient melodic structure despite not having a *Mātrkā.* An analysis along the lines pursued here in terms of pitch determination and understanding such melodic structures is currently in progress.

7. Bibliography

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Appendix

Notation for the notes is as follows:

S: *Sadaja*, **r**: *Komala Rshava*, **R**: *Shuddha Rshava*, **g**: *Komala Gāndhara*, **G**: *Shuddha Gāndhara*, **M**: *Madhyama*, **m**: *Teevra Madhyama*, **P**: *Panchama*, **d**: *Komala Dhaivata*, **D**: *Shuddha Dhaivata*, **n**: *Komala Nishāda*, **N**: *Shuddha Nishāda*

Notes in the lower Octave are indicated by a dot below the letter indicating the note. Similarly a dot above the letter indicates the note in the higher Octave. For example, **Ṛ** and **Ṙ** represent *Shuddha Rshava* in the lower and the higher Octave respectively while **R** represents the same note in the middle Octave.